

1.5 Combinations of Functions Day 1

I. Sum, Difference, Product, and Quotient Functions

1. Sum: $(f+g)(x) = f(x) + g(x)$
2. Difference: $(f-g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x)g(x)$
4. Quotient: $(f/g)(x) = f(x)/g(x)$

Ex 1) $f(x) = x^2 + 2x - 1$ and $g(x) = 2x + 1$

Find the following:

Sum:

$$x^2 + 2x - 1 + 2x + 1 = x^2 + 4x$$

Difference:

$$x^2 + 2x - 1 - (2x + 1) = x^2 - 2$$

Product:

$$(x^2 + 2x - 1)(2x + 1)$$

$$2x(x^2 + 2x - 1) + 1(x^2 + 2x - 1)$$

$$2x^3 + 4x^2 - 2x + x^2 + 2x - 1 = 2x^3 + 5x^2 - 1$$

Quotient:

$$\frac{f}{g}(x) = \frac{x^2 + 2x - 1}{2x + 1}$$

$$2x + 1 = 0$$

$$\frac{-1}{2} = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

D: all reals
except $x \neq -\frac{1}{2}$

Ex 2) Find $(f/g)(x)$ and $(g/f)(x)$ and their domains. $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4-x^2}$

$$\frac{f}{g}(x) = \frac{\sqrt{x}}{\sqrt{4-x^2}} \quad D: [0, 2)$$

D of f : $x \geq 0$ $[0, \infty)$ $(-2, 2)$

D of g : $4 - x^2 > 0$ $x^2 < 4$ $-2 < x < 2$
 $-x^2 > -4$ $x \neq \pm 2$

II. Compositions of Functions--combining 2 functions to form the composition of one with the other. $f \circ g$ read f of g

***The domain of the composition is derived from 2 places. Look at the end result and the function you are substituting in.

For example: $f(x) = x^2$ and $g(x) = x + 1$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(x+1) \\ &= (x+1)^2 \\ &= x^2 + 2x + 1 \\ D: (-\infty, \infty) \end{aligned}$$

$D: (-\infty, \infty)$

Ex 3) Find $f(g(x))$, $f(g(2))$, and $f(g(0))$.

$f(x) = \sqrt{x-1}$ and $g(x) = x-1$

$$\begin{aligned} f(g(x)) &= f(x-1) \\ &= \sqrt{x-1} \end{aligned}$$

$$\begin{aligned} f(g(0)) &= \sqrt{0-1} \\ &= \sqrt{-1} \\ &\text{D.N.E.} \end{aligned}$$

$$\begin{aligned} f(g(2)) &= \sqrt{2-1} \\ &= \sqrt{1} \\ &= 1 \end{aligned}$$

Ex 4) $f(x) = x+2$ and $g(x) = 4-x^2$. Evaluate $g(f(x))$ and when $x = 0, 1, 2$.

$$\begin{aligned} g(f(x)) &= g(x+2) \\ &= 4-(x+2)^2 \\ &= 4-(x^2+4x+4) \\ g(f(x)) &= -x^2-4x \end{aligned}$$

$$\left. \begin{array}{l} x=0: -(0)^2-4(0) = \boxed{0} \\ x=1: -(1)^2-4(1) = \boxed{-5} \\ x=2: -(2)^2-4(2) = \boxed{-12} \end{array} \right\}$$