## 1.6 Inverse Functions

A Function and its inverse will have their domains and ranges interchanged.
f: ((2, 3), (4, 6), (6, 9))

$f^{-1}$: {(3, 2), (6, 4), (9, 6)}

I. Find the inverse AND verify $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Ex 1) $f(x) = 4x$

$y = 4x$

$\frac{x}{4} = \frac{4y}{4}$

$\frac{1}{4}x = y = f^{-1}(x)$

$f(f^{-1}(x)) = f(\frac{1}{4}x) = 4(\frac{1}{4}x) = x$

$f^{-1}(f(x)) = f^{-1}(4x) = \frac{1}{4}(4x) = x$

Ex 2) $f(x) = 4x + 2$

$y = 4x + 2$

$x = 4y + 2$

$\frac{x-2}{4} = \frac{4y}{4} \Rightarrow f^{-1}(x) = \frac{1}{4}x - \frac{2}{4}$

$\frac{1}{4} \cdot \frac{2}{1}$

$f(f^{-1}(x)) = f(\frac{1}{4}x - \frac{2}{4})$

$= 4(\frac{1}{4}x - \frac{2}{4}) + 2 = x - 2 + 2 = x$

$f^{-1}(f(x)) = f^{-1}(4x+2)$

$= \frac{1}{4}(4x+2) - \frac{2}{4} = x + \frac{1}{2} - \frac{2}{4}$

$= x$

Every function has an inverse **BUT** not every inverse is a function.

III. **Horizontal Line Test**--If a horizontal line intersects the graph only once, then its inverse is a function. We can then say a function f has an inverse function $f^{-1}$ if and only if f is one-to-one.

Ex 4) Is $f(x) = \sqrt{(x + 1)}$ one-to-one? Yes

Ex 5) Is f(x) = x one-to-one? Yes

**II. Show that the functions are inverse functions of each other.**

Ex 3) $f(x) = 2x^3 - 1$ and $g(x) = \sqrt[3]{((x + 1)/2)}$

$f(g(x))$

$= 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$

$= 2\left(\frac{x+1}{2}\right) - 1$

$= x + 1 - 1$

$= x$

$g(f(x))$

$= \sqrt[3]{\frac{2x^3 - 1 + 1}{2}}$

$= \sqrt[3]{\frac{2x^3}{2}}$

$= \sqrt[3]{x^3}$

$= x$

∴ they are inverses