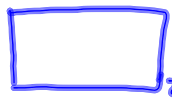
11.1 Introduction to Limits
Day 1

The notion of a **limit** is a fundamental concept of Calculus.

Ex 1) You have 24 inches of wire and are asked to form a rectangle whose area is as large as possible. What dimensions should the rectangle have?



$P = 2\ell + 2w$
$24 = 2\ell + 2w$
$\frac{24 - 2w}{2} = \frac{2\ell}{2}$
$12 - w = \ell$

$A = \ell \cdot w$
$A = (12 - w)w$
$A = 12w - w^2$
$w = 6$
$\ell = 12 - w = 6$
$A = 36$

Using Limit terminology, we can say that "the limit of A as w approaches 6 is 36 and is

written: $\lim_{w \to 6} A = \lim_{w \to 6} (12w - w^2) = 36$

Definition of a Limit: If f(x) becomes arbitrarily close to a unique L as x approaches c from either side, the limit of f(x) as x approaches c is L, written $\lim_{x \to c} f(x) = L$

Ex 2) Estimate the limit numerically. $\lim_{x \to 2} (3x - 2) = 4$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
y	3.7	3.97	3.997	4	4.003	4.03	4.3

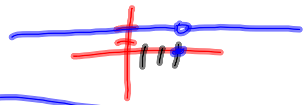
Ex 3) $\lim_{x \to 0} \frac{x}{\sqrt{(x+1)} - 1} = 2$ x=0, undefined

x	-.01	-.001	-.0001	0	0001	.001	.01
y	1.995	1.9995	1.9999	error	2.00005	2.0005	2.005

Graph and find the limit.

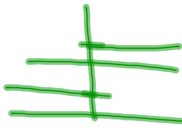
Ex 4) $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x - 1} = 2$

Ex 5) Find the limit of f(x) as x approaches 3, where f is defined as $f(x) = \begin{cases} 2, & x \neq 3 \\ 0, & x = 3 \end{cases}$



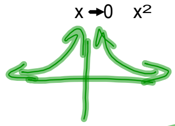
$$\lim_{x \to 3} f(x) = 2$$

Ex 6) $\lim\limits_{x \to 0} \frac{|x|}{x}$



$x > 0$: $\frac{|x|}{x} = 1$ $x < 0$: $\frac{|x|}{x} = -1$ limit D.N.E.

Ex 7) $\lim\limits_{x \to 0} \frac{1}{x^2}$



Increasing without bound

limit D.N.E.

Ex 8) $\lim\limits_{x \to 0} \sin(1/x)$

x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$
y	1	-1	1	-1

x	
y	

y values go between 1 and -1.

limit D.N.E.

Conditions under which Limits Do Not Exist

The limit of f(x) as x → c does not exist if any of the following conditions are true.

1. f(x) approaches a different number from the right side of c than it apporaches from the left side.

2. f(x) increases or decreases without bound as x approaches c.

3. f(x) oscillates between two fixed values of x approaches c.