11-2 Techniques for Evaluating Limits
Day 1

Ex 1) $\lim_{x \to -3} \dfrac{(x^2 + x - 6)}{x + 3}$

$\dfrac{(-3)^2 + -3 - 6}{-3} = \dfrac{0}{0}$

$= \dfrac{(x+3)(x-2)}{x+3} = \lim_{x \to -3} x-2 = -3-2 = \boxed{-5}$

*This works since both functions agree at all but a single number c.
*This technique should only be used when direct substitution produces zero in the numerator and denominator.

Ex 2) $\lim_{x \to 1} \dfrac{x - 1}{x^3 - x^2 + x - 1}$

$\dfrac{1-1}{(1)^3-(1)^2+1-1} = \dfrac{0}{0}$

$(x^3 - x^2) + (x-1)$

$x^2(x-1) + 1(x-1)$

$(x-1)(x^2+1)$

$\dfrac{x-1}{(x^2+1)(x-1)}$

$\lim_{x \to 1} \dfrac{1}{x^2+1}$

$= \dfrac{1}{1^2+1} = \boxed{\dfrac{1}{2}}$

II. **Rationalizing Technique:** Rationalize the numerator by multiplying the numerator and denominator by the conjugate of the numerator.

Ex 3) $\lim_{x \to 0} \dfrac{\sqrt{(x + 1)} - 1}{x} \cdot \dfrac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)} = \dfrac{0}{0}$

$= \dfrac{x+1-1}{x(\sqrt{x+1}+1)}$

$= \lim_{x \to 0} \dfrac{1}{\sqrt{x+1}+1}$

$= \dfrac{1}{\sqrt{0+1}+1} = \boxed{\dfrac{1}{2}}$

III. **Technology**: The dividing out and rationalizing techniques may not work well for finding limits of non-algebraic functions...more sophisticated analytic techniques are needed.

Ex 4) $\lim\limits_{x \to 0} (1 + x)^{1/x}$

table

$\frac{2.732 + 2.7048}{2}$

average

$\lim\limits_{x \to 0} (1+x)^{\frac{1}{x}} = 2.7184$

Ex 5) $\lim\limits_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

table

$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$