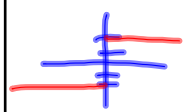
Day 2 on 11-2

One-sided Limits: $\lim_{x \to c^-} f(x) = L$ (left) $\qquad \lim_{x \to c^+} f(x) = L$ (right)

Ex 6) Find the limit as x approaches 0 from left and as x approaches 0 from the right if $f(x) = \frac{|2x|}{x}$



$$\lim_{x \to 0^-} \frac{|2x|}{x} = -2 \qquad \lim_{x \to 0^+} \frac{|2x|}{x} = 2$$

$$\lim_{x \to 0} \frac{|2x|}{x} = \text{D.N.E.}$$

Existence of a Limit

If f is a function and c and L are real numbers, then $\lim_{x \to c} f(x) = L$ if and only if both right and left limits exist and are equal to L.

Ex 7) Find the limit of f(x) as x approaches 1 if $f(x) = \begin{cases} 4 - x, & x < 1 \\ 4x - x^2, & x > 1 \end{cases}$

$$\lim_{x \to 1^-} 4 - x = 4 - 1 = \boxed{3}$$

$$\lim_{x \to 1^+} 4x - x^2 = 4(1) - (1)^2 = 4 - 1 = \boxed{3}$$

$$\lim_{x \to 1} f(x) = 3$$

Ex 8) Find the $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

A) $f(x) = 5 - 6x$

$$\lim_{h \to 0} \frac{5 - 6(x+h) - (5 - 6x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5} - \cancel{6x} - 6h - \cancel{5} + \cancel{6x}}{h}$$

$$= \lim_{h \to 0} \frac{-6\cancel{h}}{\cancel{h}} = \lim_{h \to 0} -6 = \boxed{-6}$$

$f(x) = x + 3$

$f(5) =$

B) $f(x) = \sqrt{(x - 2)}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\left(\sqrt{x+h-2} - \sqrt{x-2}\right)}{h} \frac{\left(\sqrt{x+h-2} + \sqrt{x-2}\right)}{\left(\sqrt{x+h-2} + \sqrt{x-2}\right)}$$

$$\lim_{h \to 0} \frac{\cancel{x}+h\cancel{-2}-(\cancel{x}\cancel{-2})}{h\left(\sqrt{x+h-2} + \sqrt{x-2}\right)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-2} + \sqrt{x-2}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-2} + \sqrt{x-2}}$$

$$= \frac{1}{\sqrt{x+0-2} + \sqrt{x-2}}$$

$$= \frac{1}{\sqrt{x-2} + \sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$$