11.3 The Tangent Line Problem

Day 1

Skip 33

Calculus is a branch of math that studies rates of change of functions.
(They do have applications in real life.)

*rates of change and slope...see page 763

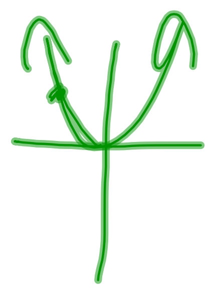
There is a more precise method which we will be doing...

Definition for slope of a graph (page 765)
The slope m of the graph of f at the point (x, f(x)) is equal to the slope of its tangent line at (x, f(x)) and is given by

$$m = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Provided the limit exists.

Ex 1) Find the slope of the graph of $f(x) = x^2$ at the point (-2, 4).



$$m_{sec} = \frac{f(-2+h) - f(-2)}{h}$$

$(h-2)^2 \rightarrow$

$$m_{sec} = \frac{(-2+h)^2 - (-2)^2}{h}$$

$$m_{sec} = \frac{h^2 - 4h + \cancel{4} - \cancel{4}}{h}$$

$$m_{sec} = \frac{h^2 - 4h}{h}$$

$$m_{sec} = \frac{\cancel{h}(h-4)}{\cancel{h}}$$

$$m_{sec} = h - 4$$

$$m_{tan} = \lim_{h \to 0} h - 4 = 0 - 4 = -4$$

Ex 2) Find the slope of f(x) = -2x + 4 using the definition.

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{-2(x+h)+4-(-2x+4)}{h}$$

$$= \frac{\cancel{-2x}-2h\cancel{+4}\cancel{+2x}\cancel{-4}}{h}$$

$$= \frac{-2\cancel{h}}{\cancel{h}} = \boxed{-2}$$

Ex 3) Find a formula for the slope of $f(x) = x^2 + 1$. What is the slope at the points (-1, 2) and (2, 5)?

$$m_{sec} = \frac{f(x+h)-f(x)}{h}$$

$$m_{sec} = \frac{(x+h)^2+1-(x^2+1)}{h} = \frac{\cancel{x^2}+2xh+h^2+\cancel{1}\cancel{-x^2}\cancel{-1}}{h}$$

$$m_{sec} = \frac{h^2+2xh}{h} = \frac{\cancel{h}(h+2x)}{\cancel{h}} = 2x+h$$

formula for slope of secant line

$$m_{tan} = \lim_{h\to 0} 2x+h = 2x+0 = 2x$$

formula for slope of tangent line.

$$(-1,2) \Rightarrow m_{tan} = 2x = 2(-1) = \boxed{-2}$$

$$(2,5) \Rightarrow m_{tan} = 2x = 2(2) = \boxed{4}$$