

Day 2 on 11-3

Definition of a Derivative: This is the formula for the slope of the tangent line to the graph of f , provided the limit exists.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \qquad \frac{dy}{dx} \qquad \frac{d}{dx}$$

Ex 1) Find the derivative of $f(x) = 3x^2 - 2x$.

$$\begin{aligned} f'(x) &= \frac{3(x+h)^2 - 2(x+h) - (3x^2 - 2x)}{h} \\ &= \frac{3(x^2 + 2xh + h^2) - 2x - 2h - 3x^2 + 2x}{h} \\ &= \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h - \cancel{3x^2} + \cancel{2x}}{h} = \frac{3h^2 + 6xh - 2h}{h} \\ &= \frac{h(3h + 6x - 2)}{h} = \lim_{h \rightarrow 0} 3h + 6x - 2 = 3(0) + 6x - 2 \\ & \qquad \qquad \qquad f'(x) = 6x - 2 \end{aligned}$$

Ex 2) $f(x) = \sqrt{x}$. Find $f'(x)$ and the slope of the graph at points $(1, 1)$ and $(4, 2)$.

$$f'(x) = \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{x+h} - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned} (1, 1) &\rightarrow \frac{1}{2\sqrt{1}} = \frac{1}{2} \\ (4, 2) &\rightarrow \frac{1}{2\sqrt{4}} = \frac{1}{4} \end{aligned}$$