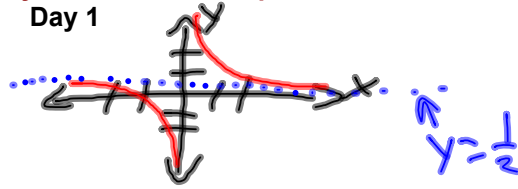


**11-4 Limits at Infinity and Limits of Sequences**

Day 1

Graph  $y = f(x) = \frac{x+1}{2x}$



$\lim_{x \rightarrow \infty} f(x) = .5$  and  $\lim_{x \rightarrow -\infty} f(x) = .5$

*These limits mean that the value of  $f(x)$  gets arbitrarily close to .5 as  $x$  decreases or increases without bound.*

Definition of Limits at Infinity

$\lim_{x \rightarrow \infty} f(x) = L_1$  and  $\lim_{x \rightarrow -\infty} f(x) = L_2$



Limits at Infinity

$\lim_{x \rightarrow \infty} \frac{1}{x^r} = 0$

$\lim_{x \rightarrow -\infty} \frac{1}{x^r} = 0$

Find the limit.

Ex 1)  $\lim_{x \rightarrow \infty} \left( 4 - \frac{3}{x^2} \right) = \lim_{x \rightarrow \infty} 4 - \lim_{x \rightarrow \infty} \frac{3}{x^2} = 4 - 0 = 4$

Ex 2)  $\lim_{x \rightarrow \infty} \frac{10}{x^2} = 0$

Ex 3)  $\lim_{x \rightarrow \infty} \frac{3+x}{3-x} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + \frac{x}{x}}{\frac{3}{x} - \frac{x}{x}}$   
 $= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + 1}{\frac{3}{x} - 1} = \frac{\lim_{x \rightarrow \infty} \frac{3}{x} + 1}{\lim_{x \rightarrow \infty} \frac{3}{x} - 1} = \frac{0 + 1}{0 - 1} = -1$

$$\text{Ex 4) } \lim_{x \rightarrow -\infty} \frac{4x-3}{2x+1}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{4x}{x} - \frac{3}{x}}{\frac{2x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{3}{x}}{2 + \frac{1}{x}} = \frac{4-0}{2+0} = \boxed{2}$$

$$\text{Ex 5) } \lim_{x \rightarrow -\infty} \frac{3x^2 - 4}{1 - x^2}$$

$$= \lim_{x \rightarrow -\infty} \frac{\frac{3x^2}{x^2} - \frac{4}{x^2}}{\frac{1}{x^2} - \frac{x^2}{x^2}} = \lim_{x \rightarrow -\infty} \frac{3 - \frac{4}{x^2}}{\frac{1}{x^2} - 1} = \frac{3-0}{0-1} = \boxed{-3}$$

$$\text{Ex 6) } \lim_{t \rightarrow \infty} \frac{1 - 2t + 6t^2}{5 + 3t - 4t^2}$$

$$= \lim_{t \rightarrow \infty} \frac{\frac{1}{t^2} - \frac{2t}{t^2} + \frac{6t^2}{t^2}}{\frac{5}{t^2} + \frac{3t}{t^2} - \frac{4t^2}{t^2}} = \lim_{t \rightarrow \infty} \frac{\frac{1}{t^2} - \frac{2}{t} + 6}{\frac{5}{t^2} + \frac{3}{t} - 4}$$

$$= \frac{0-0+6}{0+0-4} = \frac{-6}{4} = \boxed{-\frac{3}{2}}$$