11-5 The Area Problem
Day 1

Objective: Find limits of summations.
Use rectangles to approximate areas of plane regions
Use limits of summations to find areas of plane regions.

Summation Formulas and Properties (page 782)

1. $\sum_{i=1}^{n} c = c \cdot n$ c = constant

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5. $\sum_{i=1}^{n} (a_i \pm b_i) = \boxed{\sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i}$

6. $\sum_{i=1}^{n} ka_i = \boxed{k\sum_{i=1}^{n} a_i}$

Ex 1) $\sum_{i=1}^{60} 7 = 7 \cdot 60 = 420$

Ex 2) $\sum_{i=1}^{n=200} i = \frac{n(n+1)}{2} = \frac{200(200+1)}{2} = 20{,}100$

Ex 3) $\sum_{j=1}^{25} j^2 + j = \sum_{j=1}^{25} j^2 + \sum_{j=1}^{25} j$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{25(25+1)[2(25)+1]}{6} + \frac{25(25+1)}{2}$

$= 5525 + 325$

$= 5850$

Ex 4) $\sum_{i=1}^{n} \frac{i^3}{n^4} = \frac{1}{n^4}\sum_{i=1}^{n} i^3 = \frac{1}{\cancel{n^4}_{n^2}}\left[\frac{\cancel{n^2}(n+1)^2}{4}\right]$

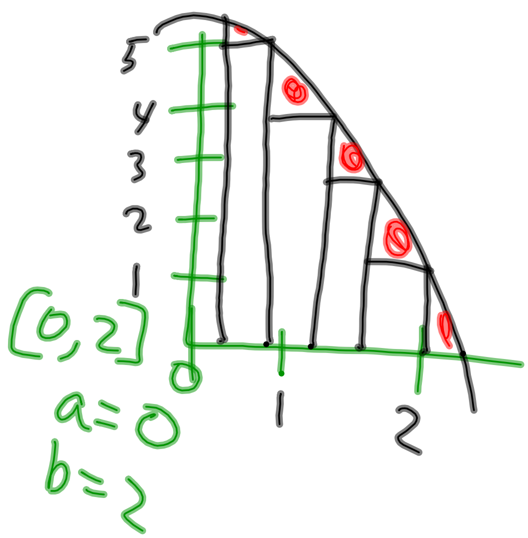
$= \frac{n^2+2n+1}{4n^2}$

a) Rewrite as a rational function.

b) Complete the table.

n	$10^0 = 1$	10^1	10^2	10^3
s(n)	1	0.3025	0.255025	0.25050025

c) Find the $\lim_{n\to\infty} s(n) = \frac{1}{4}$



Area of a Plane Region--see example 4, page 785. You can get a negative area when under x-axis, just change it to positive.

$$A = \lim_{n\to\infty} \underbrace{\sum_{i=1}^{n}}_{\text{number of rectangles}} \overbrace{f\left(a + \frac{b-a}{n} i \right)}^{\text{height}} \overbrace{\left(\frac{b-a}{n}\right)}^{\text{width}}$$

Things to remember:

f is continuous interval [a, b]

bounded by x-axis vertical lines x = a and x = b

Ex 5) See number 17 on page 788. Find the area using rectangles: $f(x) = (.25)x^3$

$[0, 2] \quad n = 8$

$a = 0 \quad b = 2$

$$\sum_{i=1}^{8} f\left(0 + \frac{(2-0)i}{8}\right)\left(\frac{2-0}{8}\right) = \sum_{i=1}^{8} f\left(\tfrac{1}{4} i\right)\left(\tfrac{1}{4}\right)$$

$$= \sum_{i=1}^{8} \frac{1}{4}\left(\frac{1}{4} i\right)^3 \left(\frac{1}{4}\right) = \sum_{i=1}^{8} \frac{1}{4}\cdot\frac{1}{64} i^3 \cdot \frac{1}{4} = \sum_{i=1}^{8} \frac{1}{1024} i^3$$

$$\frac{1}{1024}\sum_{i=1}^{8} i^3 = \frac{1}{1024}\left[\frac{n^2(n+1)^2}{4}\right]$$

$$= \frac{1}{1024}\left[\frac{8^2(8+1)^2}{4}\right]$$

$$= 1.265625$$

Ex 6) **See #21 on page 788**. Complete the table showing the approximate area of the region in the graph using n rectangles of equal width.

$[0, 3] \quad a = 0 \quad b = 3$

A) $f(x) = (1/9)x^3$

n	4	8	20	50
A	3.52	2.85	2.48	2.34

$$A = \sum_{i=1}^{n} f\left(0 + \left(\frac{3-0}{n}\right) i\right)\left(\frac{3-0}{n}\right)$$

$$= \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\cdot\left(\frac{3}{n}\right)$$

$$= \sum_{i=1}^{n} \frac{1}{9}\left(\frac{3i}{n}\right)^3 \cdot \frac{3}{n}$$

$$= \sum_{i=1}^{n} \frac{1}{\cancel{9}_{1}}\cdot\frac{\cancel{27}^{3}}{n^3} i^3 \cdot \frac{3}{n}$$

$$= \sum_{i=1}^{n} \frac{9 i^3}{n^4}$$

$$= \frac{9}{n^4}\sum_{i=1}^{n} i^3$$

$$= \frac{9}{\cancel{n^4}_{n^2}}\left[\frac{\cancel{n^2}(n+1)^2}{4}\right]$$

$$A = \frac{9(n+1)^2}{4n^2}$$

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