2.3 Real Zeros of Polynomial Functions Day 1

I. Long Division

Ex 1) $(6x^3 - 19x^2 + 16x - 4) \ / \ (x - 2)$

$$\boxed{6x^2 - 7x + 2}$$

$x - 2 \,\big)\, 6x^3 - 19x^2 + 16x - 4$

$-6x^3 + 12x^2$

$-7x^2 + 16x$

$+7x^2 - 14x$

$2x - 4$

$-2x + 4$

0

Ex 2) $(x^5 + 3x^3 - 4) \ / \ (x^2 + x - 1)$

$$x^3 - x^2 + 5x - 6 + \frac{11x - 10}{x^2 + x - 1}$$

$x^2 + x - 1 \,\big)\, x^5 + 0x^4 + 3x^3 + 0x^2 + 0x - 4$

$-x^5 - x^4 + x^3$

$-x^4 + 4x^3 + 0x^2$

$+x^4 + x^3 - x^2$

$5x^3 - x^2 + 0x$

$-5x^3 - 5x^2 + 5x$

$-6x^2 + 5x - 4$

$+6x^2 + 6x - 6$

$11x - 10$

II. Synthetic Division--short cut, only works when dividing by x - k.

Ex 3) $(x^4 - 10x^2 - 2x + 4) / (x + 3)$

$$\begin{array}{r|rrrr|r} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array}$$

1 - remainder

$x^3 - 3x^2 - x + 1 + \frac{1}{x+3}$

Ex 4) $(2x^3 + 14x^2 - 20x + 7) / (x + 6)$

$$\begin{array}{r|rrr|r} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$

$2x^2 + 2x - 32 + \frac{199}{x+6}$

Remainder Theorem: If a polynomial is divided by x - k, the remainder is r = f(k).

Ex 5) Use the remainder theorem to evaluate when x = -2 if

$f(x) = 3x^3 + 8x^2 + 5x - 7.$

$$\begin{array}{r|rrr|r} -2 & 3 & 8 & 5 & -7 \\ & & -6 & -4 & -2 \\ \hline & 3 & 2 & 1 & -9 \end{array}$$

remainder

$f(-2) = -9$

$f(-2) = 3(-2)^3 + 8(-2)^2 + 5(-2) - 7$
$= 3(-8) + 8(4) + -10 - 7$
$= -24 + 32 - 17$
$= 8 - 17$
$= -9$

Factor Theorem: A polynomial f(x) has a factor (x - k) if and only if f(k)= 0.

Ex 6) Show that (x - 2) and (x + 3) are factors of
$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$.

$$
\begin{array}{r|rrrr|r}
2 & 2 & 7 & -4 & -27 & -18 \\
 & & 4 & 22 & 36 & 18 \\
\hline
 & 2 & 11 & 18 & 9 & 0
\end{array}
$$

remainder

$$
\begin{array}{r|rrrr|r}
-3 & 2 & 7 & -4 & -27 & -18 \\
 & & -6 & -3 & 21 & 18 \\
\hline
 & 2 & 1 & -7 & -6 & 0
\end{array}
$$

Ex 7) $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$, factors are (x - 5) and (x + 4)