2-5 The Fundamental Theorem of Algebra

Fundamental Theorem of Algebra P. 135

If f(x) is a polynomial of degree n, where n>0, then f has at least one zero in the complex number system (which means reals and nonreals.

I . Find all the zeros of the polynomial and write the linear factorization.

Ex 1) $f(x) = x^2 - 6x + 9$

$(x-3)(x-3)$

$(x-3)(x-3)=0$

$x-3=0 \quad x-3=0$

$x=3$ repeated

Ex 2) $f(x) = x^3 + 4x$

$x(x^2+4)=0$

$x=0 \qquad x^2+4=0$

$\sqrt{x^2}=\sqrt{-4}$

$x=\pm 2i$

$x(x^2+4)$

$x(x-2i)(x+2i)$

$-2i \cdot 2i = -4i^2$

$+4$

- - + + +

Ex 3) $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ (Graph and find all rational zeros first.)

pos 2 or 0
neg. 1

	1	0	1	2	-12	8
-2	1	-2	5	-8	4	0
1	1	-1	4	-4	0	
1	1	0	4	0		

$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$x = -2$

$x = 1$

$x = 1$

$x = \pm 2i$

$(x+2)(x-1)(x-1)(x+2i)(x-2i)$

Ex 4) $f(x) = x^2 + 6x - 2$

$a = 1$

$b = 6$

$c = -2$

$36 - -8$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{44}}{2} \Rightarrow \sqrt{4} \cdot \sqrt{11} = 2\sqrt{11}$

$x = \frac{-6 \pm 2\sqrt{11}}{2}$

$x = -3 \pm \sqrt{11}$

$[x - (-3 + \sqrt{11})][x - (-3 - \sqrt{11})]$

$(x + 3 - \sqrt{11})(x + 3 + \sqrt{11})$