

Day 2 on 2.5

Find a polynomial with the given zeros.

Ex 1) -1, -1, 3i, -3i

$$(x+1)(x+1)(x-3i)(x+3i)$$

$$(x^2+2x+1)(x^2-9i^2) = (x^2+2x+1)(x^2+9)$$

$$x^2(x^2+2x+1) + 9(x^2+2x+1)$$

$$= x^4 + 2x^3 + x^2 + 9x^2 + 18x + 9 = \boxed{x^4 + 2x^3 + 10x^2 + 18x + 9}$$

Ex 2) $f(x) = x^2 + 1$ is said to be irreducible over the reals and is prime. But we can still write a linear factorization.

$$(x+i)(x-i)$$

$$x^2 - 2$$

$$(x-\sqrt{2})(x+\sqrt{2})$$

Ex 3) Use the given zero to find all the zeros of the polynomial.

$$f(x) = x^3 + x^2 + 9x + 9$$

zero: 3i, -3i

$$x = -1$$

$$\begin{array}{r|rrrr} 3i & 1 & 1 & 9 & 9 \\ & & 3i & 3i+9i^2 & 9i^2 \\ \hline & 1 & 1+3i & 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3i & 1 & 1+3i & 3i \\ & & -3i & -3i \\ \hline & 1 & 1 & 0 \\ & x+1=0 & & \end{array}$$

$$x = -1$$

Ex 4) Use the given zero to find all the zeros of the polynomial.

$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

zero: $1 + 3i, 1 - 3i, 3, -2$

$$\begin{array}{r|rrrr|r} 1+3i & 1 & -3 & 6 & 2 & -60 \\ & 1+3i & -1+3i & 4+18i & & 60 \\ \hline & 1 & -2+3i & -5-3i & 6-16i & 0 \end{array}$$

$(1+3i)(6-16i) = 6-16i+18i-48i^2 = 6-54i+2+48 = 54-52i$

$$\begin{array}{l} (1+3i)(-2+3i) \\ = 3i-6i-2-9i^2 \\ = -2-3i+9 \\ = 7-3i \\ \hline (1+3i)(-5-3i) \\ = -5-18i-9i^2 \\ = -5-18i+9 \\ = 4-18i \end{array}$$

$$\begin{array}{r|rrrr|r} 1-3i & 1 & -2+3i & -5-3i & 6-16i \\ & 1-3i & -1+3i & -6+18i & & 6-16i \\ \hline & 1 & -1 & -6 & 0 & 0 \end{array}$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x-3=0 \quad x+2=0$$

$$\boxed{x=3} \quad \boxed{x=-2}$$