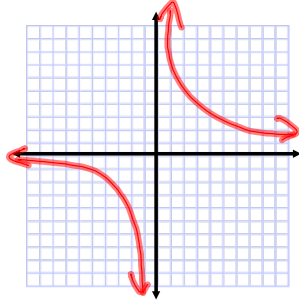


2.6 Rational Functions & Asymptotes Day 1

A Rational Function can be written in the form: $f(x) = \frac{N(x)}{D(x)}$
 *N(x) and D(x) are polynomials where $D(x) \neq 0$

Ex1) Graph $f(x) = \frac{1}{x}$ and find:



Domain: $(-\infty, 0) \cup (0, \infty)$
 Range: $(-\infty, 0) \cup (0, \infty)$
 Intercepts: none
 Increasing: nowhere
 Decreasing: $(-\infty, 0) \cup (0, \infty)$
 Even/Odd: odd
 Vertical Asymptote: $x=0$
 Horizontal Asymptote: $y=0$

Asymptotes:

- 1) Vertical: Zeros of the denominator
- 2) Horizontal: $n = \text{degree of numerator}, m = \text{degree of denominator}$
 - a) $n < m$; $y = 0$ (the axis) is a horizontal asymptote
 - b) $n = m$; $y = \frac{a_n}{b_n}$ is a horizontal asymptote
 $a_n = \text{leading coefficient of numerator}$
 $b_n = \text{leading coefficient of denominator}$
 - c) $n > m$; no horizontal asymptote

Find the domain, vertical and horizontal asymptotes. (Graph and Check)

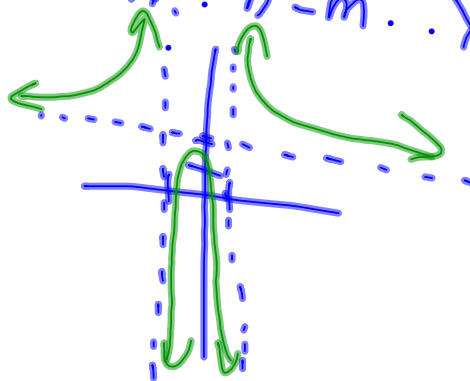
Ex2) $f(x) = \frac{2x^2}{x^2-1}$

$x^2 - 1 = 0$
 $\sqrt{x^2} = \sqrt{1}$
 $x = \pm 1$

D: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

V.A.: $x = 1$ and $x = -1$

H.A.: $n = m \therefore y = \frac{2}{1} \Rightarrow y = 2$



Find the domain, vertical and horizontal asymptotes. (Graph and Check)

Ex3) $f(x) = \frac{2x}{3x^2+1}$

$$3x^2 + 1 = 0$$

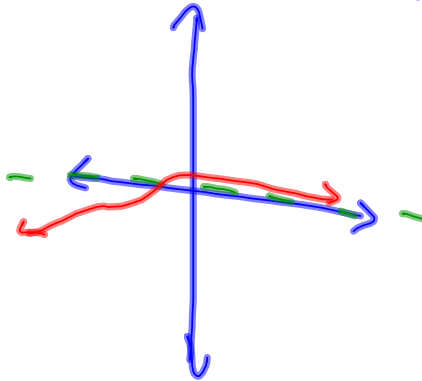
$$3x^2 = -1$$

$$x^2 = \sqrt{-\frac{1}{3}}$$

$D: (-\infty, \infty)$

V.A.: none

H.A.: $n < m \therefore y = 0$



Find the domain, vertical and horizontal asymptotes. (Graph and Check)

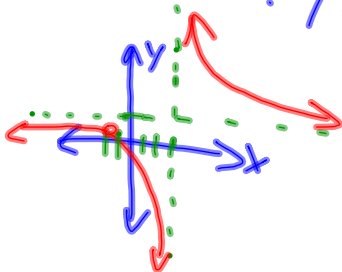
Ex4) $f(x) = \frac{x^2 + x - 2}{x^2 - x - 6} = \frac{\overset{\text{holes}}{\cancel{(x+2)}}(x-1)}{(\cancel{x+2})(x-3)} = \frac{x-1}{x-3}$

$D: (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

V.A.: $x = 3$

H.A.: $n = m \therefore y = \frac{1}{1} \Rightarrow y = 1$

hole at $x = -2$
cancel = holes



Find the domain, vertical and horizontal asymptotes. (Graph and Check)

$$\text{Ex5) } f(x) = \frac{3x^3 + 7x^2 + 2}{-4x^3 + 5}$$

$$\begin{aligned} -4x^3 + 5 &= 0 \\ -4x^3 &= -5 \\ \sqrt[3]{-4x^3} &= \sqrt[3]{-5} \\ x &= \sqrt[3]{\frac{5}{4}} \end{aligned}$$

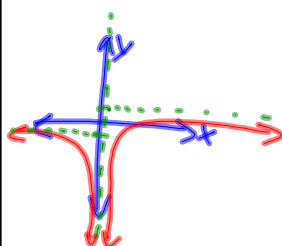
$$D: (-\infty, \sqrt[3]{\frac{5}{4}}) \cup (\sqrt[3]{\frac{5}{4}}, \infty)$$

$$V.A.: x = \sqrt[3]{\frac{5}{4}}$$

$$H.A.: n = m \therefore y = -\frac{3}{4}$$

Find the domain, vertical and horizontal asymptotes. (Graph and Check)

$$\text{Ex6) } f(x) = \frac{x-3}{|x|}$$



$$D: (-\infty, 0) \cup (0, \infty)$$

$$V.A.: x = 0$$

$$H.A.: n = m$$

$$y = 1 \text{ to right}$$

$$y = -1 \text{ to left}$$