3-2 Logarithm Functions and Their Graphs

An exponential function ($f(x) = a^x$, $a>0$, $a\neq 1$) passes the horizontal line test, therefore must have an inverse function...which is called a logarithmic function with base a.

Log Function: For $x > 0$, $a>0$, $a \neq 1$

$y = \log_a x$ if and only if $a^y = x$

exponential

Properties

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = 1$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$
4. If $\log_a x = \log_a y$, then $x = y$.

$\log_3 3^4 = 4$

Natural Log: For $x > 0$, $y = \ln x$ if and only if $e^y = x$.

$y = \log_e x$

Properties:

1. $\ln 1 = 0$ because $e^0 = 1$
2. $\ln e = 1$ because $e^1 = e$
3. $\ln e^x = x$ and $e^{\ln x} = x$
4. If $\ln x = \ln y$, then $x = y$.

I. Write the log equation in exponential form.

Ex 1) $\log_3 81 = 4$ $\Rightarrow$ $3^4 = 81$

Ex 2) $\log_7(1/49) = -2$ $\Rightarrow$ $7^{-2} = \frac{1}{49}$

Ex 3) $\ln 1 = 0$ $\Rightarrow$ $e^0 = 1$

$\log_e 1 = 0$

II. Write the exponential equation to log form.

Ex 4) $8^2 = 64$ $\Rightarrow$ $\log_8 64 = 2$

Ex 5) $e^x = 4$ $\Rightarrow$ $\log_e 4 = x$ $\Rightarrow$ $\ln 4 = x$

III. Evaluate without a calculator.

Ex 6) $f(x) = \log_3 x$, $x = 1$

$\log_3 1 = 0$

Ex 7) $f(x) = \log_4 x$, $x = 2$

$$\log_4 2 = y$$
$$4^y = 2$$
$$(2^2)^y = 2$$
$$2^{2y} = 2^1$$
$$2y = 1$$
$$\boxed{y = \frac{1}{2}}$$
$$\therefore \log_4 2 = \frac{1}{2}$$

Ex 8) $f(x) = \log_{16} x$, $x = 1/4$

$$\log_{16} \tfrac{1}{4} = y$$
$$16^y = \frac{1}{4}$$
$$(4^2)^y = 4^{-1}$$
$$4^{2y} = 4^{-1}$$
$$2y = -1$$
$$\boxed{y = -\frac{1}{2}}$$

IV. Use a calculator to evaluate.

Ex 9) $f(x) = \log_{10} x$, $x = 4/5$

$$\log_{10}\left(\frac{4}{5}\right) = -0.0969$$

V. Solve for x.

Ex 10) $\cancel{\log_7} x = \cancel{\log_7} 9$

$$x = 9$$

Ex 11) $\log_6 6^2 = x$

$$2 = x$$