3.3 Properties of Logarithms

I. Change of base formula:

$\log_a x = \dfrac{\log_{10} x}{\log_{10} a} = \dfrac{\ln x}{\ln a}$

Ex 1) $\log_4 25 = \dfrac{\log 25}{\log 4} \approx 2.3219$ $\qquad \dfrac{\ln 25}{\ln 4} \approx 2.3219$

Ex 2) $\log_2 12 = \dfrac{\log 12}{\log 2} \approx 3.5850$

II. Properties of Logarithms

1. $\log_a (uv) = \log_a u + \log_a v$
2. $\log_a (u/v) = \log_a u - \log_a v$
3. $\log_a u^n = n\log_a u$

Ex 3) Rewrite and simplify using the properties.

A) $\log_4 8$

$= \log_4 2^3$

$= 3\log_4 2$

$= 3\log_4 4^{\frac{1}{2}}$

$= 3 \cdot \frac{1}{2}$

$= \frac{3}{2}$

B) $\ln (2/27)$

$\ln 2 - \ln 27$

≈ -2.6027

Ex 4) **Use the properties of logs to expand the expression as a sum, difference, and/or multiple logs**.

A) $\log_4 (5x^3y)$ $= \log_4 5 + \log_4 x^3 + \log_4 y$

$= \log_4 5 + 3\log_4 x + \log_4 y$

B) $\ln\left(\frac{\sqrt{3x-5}}{7}\right)$ $= \ln\sqrt{3x-5} - \ln 7$

$= \ln (3x-5)^{\frac{1}{2}} - \ln 7$

$= \frac{1}{2}\ln(3x-5) - \ln 7$

C.) $\ln\sqrt[3]{\frac{x}{y}}$ $= \ln\sqrt[3]{x} - \ln\sqrt[3]{y}$ $\qquad \sqrt[3]{x} = x^{\frac{1}{3}}$

$= \ln x^{\frac{1}{3}} - \ln y^{\frac{1}{3}}$

$= \frac{1}{3}\ln x - \frac{1}{3}\ln y$

D.) $\log_b \frac{\sqrt{x}\cdot y^4}{z^4}$ $= \log_b \sqrt{x} + \log_b y^4 - \log_b z^4$

$= \frac{1}{2}\log_b x + 4\log_b y - 4\log_b z$