3.4 Solving Exponential and Logarithmic Equations

One-to-One Properties

$a^x = a^y$ if and only if $x = y$

$\log_a x = \log_a y$ if and only if $x = y$

Be sure to check for extraneous solutions

Inverse Properties

$a^{\log_a y} = y$ $\quad 5^{\log_5 3} = 3$

$\log_a a^x = x$

I. Solve Simple Equations...solving for x.

Ex 1) a) $2^x = 32$

$2^x = 2^5$

$x = 5$

b) $\ln x - \ln 3 = 0$

$+\ln 3 \quad +\ln 3$

$\ln x = \ln 3$

$x = 3$

c) $(1/3)^x = 9$

$(3^{-1})^x = 3^2$

$3^{-x} = 3^2$ $\Rightarrow$ $-x = 2$

$x = -2$

$\ln e = 1$

$\log_e e^1$

$\ln e^3 = 3$

d) $e^x = 7$

$\ln e^x = \ln 7$

$x = \ln 7$

$e^{\ln 7} = 7$

$e^{\log_e 7} = 7$

$7 = 7$ ✓

e) $\ln x = -3$

① $e^{\ln x} = e^{-3}$

$x = e^{-3}$

② $\log_e x = -3$

$x = e^{-3}$

f) $\log_{10} x = -1$

$x = 10^{-1}$

Ex 2) $\ln(2x - 1) = 5$

$2x - 1 = e^5$

$\frac{2x}{2} = \frac{1 + e^5}{2}$

$x = \frac{1 + e^5}{2}$

Ex 3) $\frac{4e^{2x}}{4} = \frac{40}{4}$

$e^{2x} = 10$

$\ln e^{2x} = \ln 10$

$\frac{2x}{2} = \frac{\ln 10}{2}$

$x = \frac{\ln 10}{2}$

$x \approx 1.151$

Ex 4) $5^{-t/2} = .2$

$5^{-\frac{1}{2}t} = 0.2$

$5^{-\frac{1}{2}t} = 5^{-1}$

$(-2)\ -\frac{1}{2}t = -1(-2)$

$t = 2$

$\log 5^{-\frac{1}{2}t} = \log 0.2$

$(-2)\ -\frac{1}{2}t \log 5 = \log 0.2^{(-2)}$

$\frac{t \log 5}{\log 5} = \frac{-2 \log 0.2}{\log 5}$

$t \approx 2$

Ex 5) $e^{2x} - 5e^{x} + 6 = 0$

$(e^x - 3)(e^x - 2) = 0$

$e^x - 3 = 0$

$e^x = 3$

$\ln e^x = \ln 3$

$x = \ln 3$

$e^x - 2 = 0$

$e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$