3.5 Exponential and Logarithmic Models

Most of the story problems will have equations written for you.
Those that do not will most likely be an exponential growth or decay type problem and you will use the following formulas.

Exponential Growth Model: $y = ae^{bx}$, $b > 0$
Exponential Decay Model: $y = ae^{-bx}$, $b > 0$

Ex 1) **Population Growth**: Estimates of the world population (in millions) from 1995 - 2004 are shown in the table. An exponential model that approximates this data is given by $p = 5344e^{.012744t}$, $5 \leq t \leq 14$, where p is the population (in millions) and t = 5 represents 1995. Compare the values given by the model with the estimates shown in the table. According to this model, when will the population reach 6.8 billion?

year	population
1995	5685
1996	5764
1997	5844
1998	5923
1999	6002
2000	6079
2001	6154
2002	6228
2003	6302
2004	6376

Ex 2) **Modeling Population Growth**: In a research experiment, Mr. Johannes' population of fruit flies is increasing according to the law of exponential growth. After 2 days, there are 100 flies, and after 4 days, there are 300 flies. How many flies will there be after 5 days?

$y = ae^{bt}$

$\frac{100}{e^{2b}} = \frac{ae^{2b}}{e^{2b}}$

$a = \frac{100}{e^{2b}}$

$\left(\frac{1}{100}\right) 300 = \left(\frac{100}{e^{2b}}\right) e^{4b} \left(\frac{1}{100}\right)$

$3 = \frac{e^{4b}}{e^{2b}}$

$\ln 3 = \ln e^{2b}$

$\ln 3 = 2b$

$\frac{\ln 3}{2} = b$

$300 = ae^{4b}$

$a = \frac{100}{e^{2\left(\frac{\ln 3}{2}\right)}}$

$a = 33.\overline{3}$

$y = 33.\overline{3}e^{0.5493t}$

$y = 33.\overline{3}e^{0.5493(5)}$

$y \approx 520$ flies

Ex 3) **Carbon Dating:** In living organic material, the ratio of the content of radioactive carbon isotopes (carbon 14) to the content of nonradioactive carbon isotopes (carbon 12) is about 1 to 10^{12}. When organic material dies, its carbon 12 content remains fixed, whereas its radioactive carbon 14 begins to decay with a half-life of 5730 years. To estimate the age of dead organic material, scientists use the following formula, which denotes the ratio of carbon 14 to carbon 12 present at any time t (in years).

$$r = \frac{1}{10^{12}} e^{-t/8267}$$

The ratio of carbon 14 to carbon 12 in a newly discovered fossil is $r = \frac{1}{10^{13}}$.

Estimate the age of the fossil.

$$(10^{12})\frac{1}{10^{13}} = \frac{1}{\cancel{10^{12}}} e^{-\frac{t}{8267}} \cancel{(10^{12})}$$

$$\ln \frac{1}{10} = \ln e^{-\frac{t}{8267}}$$

$$(8267)\ln \frac{1}{10} = \frac{-t}{\cancel{8267}}(\cancel{8267})$$

$$-19{,}035 \approx -t$$

$t \approx 19{,}035$ years old

Ex 4) **Spread of a virus**: On a college campus of 5000 students, one student returns from vacation with a contagious flu virus. The spread of the virus is modeled by

$$y = \frac{5000}{1 + 4999e^{-.8t}} \qquad t \geq 0$$

where y is the total number infected after t days.
The college will cancel classes when 40% or more of the students are infected.

a) How many students are infected after 5 days?

$$y = \frac{5000}{1+4999e^{-0.8(5)}} \approx 54 \text{ students}$$

b) After how many days will the college cancel classes?

$$0.4 * 5000 = 2000$$

$$(1+4999e^{-0.8t})\, 2000 = \frac{5000}{\cancel{1+4999e^{-0.8t}}} \cancel{(1+4999e^{-0.8t})}$$

$$\frac{(1+4999e^{-0.8t})2000}{2000} = \frac{5000}{2000}$$

$$1+4999e^{-0.8t} = 2.5$$
$$-1 \qquad\qquad -1$$

$$\frac{4999e^{-0.8t}}{4999} = \frac{1.5}{4999}$$

$$\ln e^{-0.8t} = \ln \frac{1.5}{4999}$$

$$\frac{-0.8t}{-0.8} = \frac{\ln \frac{1.5}{4999}}{-0.8}$$

$t \approx 10.139$

$t \approx 10$ days

Ex 5) On the Richter Scale, the magnitude R of an earthquake of intensity I is given by $R = \log_{10} (I/I_o)$ where $I_o = 1$ is the minimum intensity used for comparison. Intensity is a measure of the wave energy of an earthquake.

In 2001, the coast of Peru experienced an earthquake that measured 8.4 on the Richter scale. In 2003, Colima, Mexico experienced an earthquake that measured 7.6 on the Richter scale. Find the intensity of each earthquake and compare the two intensities.

$8.4 = \log_{10} \frac{I}{1}$

$10^{8.4} = I$

$I \approx 251,188,643.2$

$7.6 = \log_{10} \frac{I}{1}$

$10^{7.6} = I$

$I \approx 39,810,717.06$

$$\frac{251,188,643.2}{39,810,717.06} \approx 6.3$$

6 times more intense