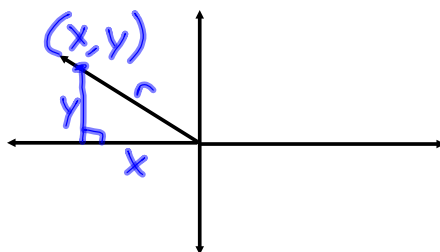4.4 Trigonometric Functions of Any Angle Day 1

I. **Trig Functions of Any Angle**: θ is an angle in standard position with (x, y) on a point on the terminal side of θ .

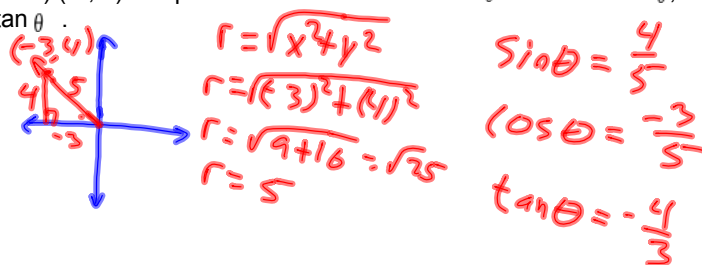$r = \sqrt{x^2 + y^2}$

(x, y)

$\sin\theta = y/r$

$\cos\theta = x/r$
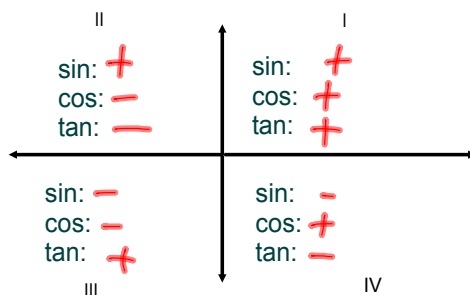
$\tan\theta = y/x$

$\csc\theta = r/y$

$\sec\theta = r/x$

$\cot\theta = x/y$

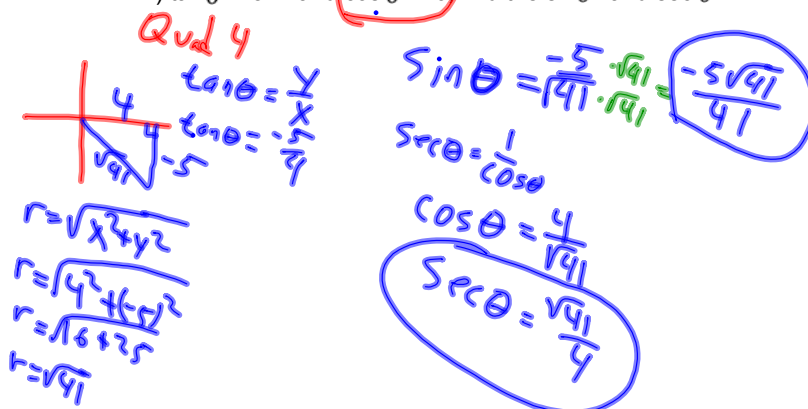Ex 1) (-3, 4) is a point on the terminal side of θ . Find the $\sin\theta$, $\cos\theta$, and $\tan\theta$.
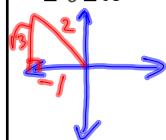
$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + (4)^2}$

$r = \sqrt{9 + 16} = \sqrt{25}$

$r = 5$

$\sin\theta = \dfrac{4}{5}$

$\cos\theta = \dfrac{-3}{5}$

$\tan\theta = -\dfrac{4}{3}$

$\sin = y$
$\cos = x$

II
sin: $+$
cos: $-$
tan: $-$

I
sin: $+$
cos: $+$
tan: $+$

III
sin: $-$
cos: $-$
tan: $+$

IV
sin: $-$
cos: $+$
tan: $-$

Ex 2) $\tan\theta = -5/4$ and $\cos\theta > 0$ find the $\sin\theta$ and $\sec\theta$.

Qud 4

$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{-5}{4}$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{4^2 + (-5)^2}$

$r = \sqrt{16 + 25}$

$r = \sqrt{41}$

$\sin\theta = \dfrac{-5}{\sqrt{41}} \cdot \dfrac{\sqrt{41}}{\sqrt{41}} = \dfrac{-5\sqrt{41}}{41}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cos\theta = \dfrac{4}{\sqrt{41}}$

$\sec\theta = \dfrac{\sqrt{41}}{4}$

Ex 3) Find the values of the 6 trig functions if sec θ= -2 and
$0 \le \theta \le \pi$.   Quad II

$\sec\theta = \frac{r}{x}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\sec\theta = \frac{-2}{1} = \frac{2}{-1} = -1$

$\cos\theta = \frac{-1}{2}$ $\csc\theta = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$x = -1$
$y = \sqrt{3}$ $\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$ $\cot\theta = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$
$r = 2$

II. <u>Quadrantel Angles</u>: are angles that have their terminal side on the x or y-axis.
Ex 4) Find the sin of 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$.
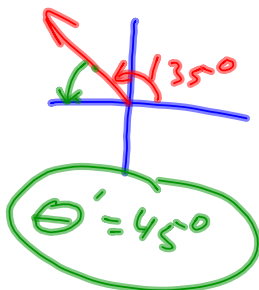
$\sin 0 = 0$ $\sin \pi = 0$
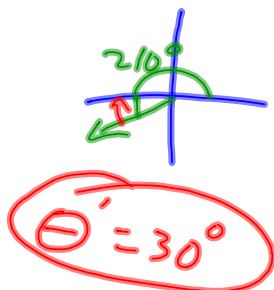
$\sin \frac{\pi}{2} = 1$ $\sin \frac{3\pi}{2} = -1$

III. <u>Reference Angles</u>--The values of the trig functions of angles greater than 90° (or less than 90°) can be determined from their values at corresponding acute angles called reference angles.
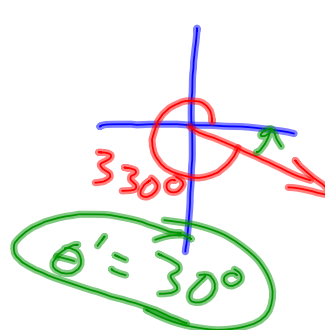
Ex 5) $\theta = 135°$ Ex 6) θ 210 °   Ex 7) θ 330 °

$\theta' = 45°$ $\theta' = 30°$ $\theta' = 30°$

Ex 8) Evaluate the sine, cosine, and tangent of angle without a calculator.

A) $300°$

$300°$

$\theta' = 60°$

$\sin 300° = \sin 60° =$

$\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{-\frac{\sqrt{3}}{2} \cdot 2}{\frac{1}{2} \cdot 2} = -\sqrt{3}$

B) $\frac{4\pi}{3}$

$240°$

$60°$

$\sin 60° = -\frac{\sqrt{3}}{2}$

$\cos 60° = -\frac{1}{2}$

$\tan 60° = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

1.) Find reference angle

2.) Find trig. value of reference angle

3.) Is it positive or negative h that quadrant.