

4.7 Inverse Trigonometric Functions

I. Graph $y = \sin x$. Explain what its inverse would be and why.

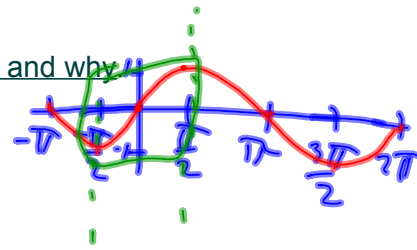
$$y = \arcsin x = \sin^{-1} x$$

$$y = \arccos x = \cos^{-1} x$$

$$y = \arctan x = \tan^{-1} x$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin^{-1} \left(\frac{\sqrt{2}}{2} \right) = \frac{\pi}{4}$$



Function

$y = \arcsin x$ if and only if $\sin y = x$

$y = \arccos x$ if and only if $\cos y = x$

$y = \arctan x$ if and only if $\tan y = x$

Domain

$$-1 \leq x \leq 1$$

$$-1 \leq x \leq 1$$

$$-\infty < x < \infty$$

Range

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$0 \leq y \leq \pi$$

$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$



II. Find the exact value without a calculator.

a) $\arcsin(-.5) = -\frac{\pi}{6}$ or -30°

b) $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$

$$\frac{\pi}{3}$$
 or 60°

c) $\arccos \left(\frac{\sqrt{2}}{2} \right)$

$$\frac{\pi}{4}$$
 or 45°

d) $\cos^{-1}(-1)$

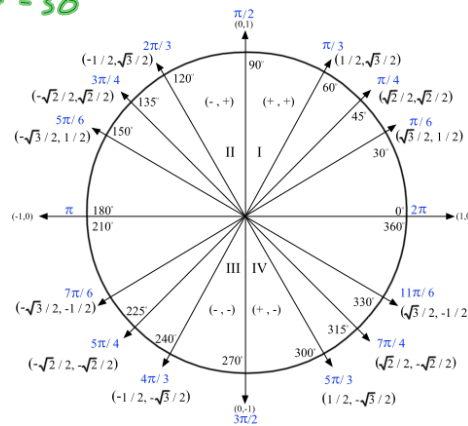
$$\pi$$
 or 180°

e) $\arctan 0$

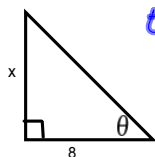
$$0^\circ$$

f) $\sin^{-1}(2)$ = undefined

g) $\tan^{-1}(-1) = -\frac{\pi}{4}$ or -45°



III. Use an inverse trig function to write θ as a function of x .



$$\tan^{-1} \tan \theta = \frac{x}{8} \tan^{-1}$$

$$\theta = \tan^{-1} \left(\frac{x}{8} \right)$$

IV. Write an algebraic expression that is equivalent to the expression.

$\sin(\arccos 3x)$

$\cos \arccos 3x = \cos \angle$
 $\frac{3x}{1} = \cos \angle$

$\sin \theta = \frac{\sqrt{1-9x^2}}{1}$

$(3x)^2 + b^2 = 1^2$
 $9x^2 + b^2 = 1$
 $b^2 = 1 - 9x^2$
 $b = \sqrt{1-9x^2}$

V. Find the exact value of the expression.

$\cos(\arcsin 24/25)$