6-2 Law of Cosines
Day 1

I. Use when given:
1) three sides (SSS)
2) two sides and the included angle (SAS)

Law of Cosines:
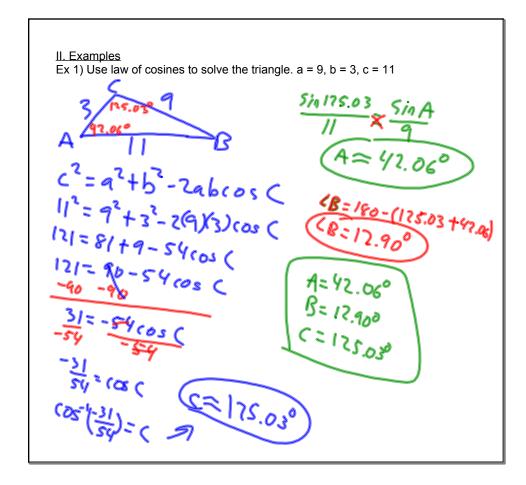
Standard Form   Alternate Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab}$$

II. Examples
Ex 1) Use law of cosines to solve the triangle. a = 9, b = 3, c = 11

$$\frac{\sin 125.03}{11} = \frac{\sin A}{9}$$

$$A \approx 42.06°$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$11^2 = 9^2 + 3^2 - 2(9)(3)\cos C$$
$$121 = 81 + 9 - 54\cos C$$
$$121 = 90 - 54\cos C$$
$$-90 \quad -90$$
$$\frac{31}{-54} = \frac{-54\cos C}{-54}$$
$$\frac{-31}{54} = \cos C$$
$$\cos^{-1}\left(\frac{-31}{54}\right) = C$$

$$C \approx 125.03°$$

∠B = 180 - (125.03 + 42.06)
∠B = 12.90°

A = 42.06°
B = 12.90°
C = 125.03°

Ex 2) Solve the given triangle if A = 115°, c = 10 cm, b = 15 cm

$$\frac{\sin 115}{21.26} = \frac{\sin B}{15}$$

$$B = 39.75°$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 15^2 + 10^2 - 2(15)(10) \cos 115°$$

$$a^2 = 225 + 100 - 300 \cos 115°$$

$$a^2 = \sqrt{451.7854785}$$

$$a \approx 21.26$$

$$C = 180 - (39.75 + 115)$$

$$C \approx 25.25°$$

$$a = 21.26$$
$$B = 39.75°$$
$$C = 25.25°$$

III. <u>Heron's Formula</u>

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = the semi-perimete

$$= \frac{a + b + c}{2}$$

Ex 3) Find the area if a = 5, b = 8, and c = 10.

$$S = \frac{5+8+10}{2} = \frac{23}{2} = 11.5$$

$$A = \sqrt{11.5(11.5-5)(11.5-8)(11.5-10)}$$

$$A = \sqrt{11.5(6.5)(3.5)(1.5)}$$

$$A = 19.81$$

Remember: Now you have 3 area formulas!
1. A = .5bh
2. A = .5bcsin A = .5acsin B = .5absin C
3. Heron's...see above