

6-3 Vectors in the Plane

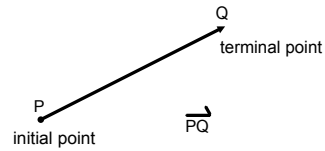
Day 1

I. **Directed Line Segments:** read page 408

Directed Line Segments have

1. magnitude = length =
2. direction

Bold lowercase letters (**u**, **v**, and **w**) represent vectors



II. **Equivalent Directed Line Segments**

Ex 1) let **u** represent \vec{PQ} if $P = (0, 0)$ and $Q = (3, 2)$

let **v** represent \vec{RS} if $R = (1, 2)$ and $S = (4, 4)$

Show that $u = v$.

$$\begin{aligned} \|\vec{PQ}\| &= \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{9+4} = \sqrt{13} \\ \|\vec{RS}\| &= \sqrt{(4-1)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13} \end{aligned}$$

Same magnitude

$$m_{\vec{PQ}} = \frac{\Delta y}{\Delta x} = \frac{2-0}{3-0} = \frac{2}{3}$$

$$m_{\vec{RS}} = \frac{\Delta y}{\Delta x} = \frac{4-2}{4-1} = \frac{2}{3}$$

Same slopes, so $u = v$

III. **Component Form:** initial point $P = (p_1, p_2)$, terminal point $Q = (q_1, q_2)$

$$\vec{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle \quad (\text{must start with } q)$$

Ex 2) Find the component form and magnitude of the vector.

A)

terminal → initial

$$\vec{PQ} = \langle -1-4, 5-(-7) \rangle$$

$$\vec{PQ} = \langle -5, 12 \rangle$$

$$\|\vec{PQ}\| = \sqrt{(-5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = 13$$

B)

$$\vec{PQ} = \langle 4-0, -2-0 \rangle$$

$$= \langle 4, -2 \rangle$$

$$\|\vec{PQ}\| = \sqrt{(4)^2 + (-2)^2}$$

$$= \sqrt{16+4}$$

$$= \sqrt{20}$$

$$= \sqrt{4 \cdot 5}$$

$$= 2\sqrt{5}$$

III. **Addition/Scalar Multiplication:** Page 410 and Properties on page 412

Ex 3) $u = \langle 2, 8 \rangle$ and $v = \langle -3, 4 \rangle$

Find:

$$a) u + v = \langle 2 + (-3), 8 + 4 \rangle = \langle -1, 12 \rangle$$

$$b) u - v = \langle 2 - (-3), 8 - 4 \rangle = \langle 5, 4 \rangle$$

c) $3u + 5v$

$$3u = \langle 3 \cdot 2, 3 \cdot 8 \rangle = \langle 6, 24 \rangle$$

$$5v = \langle 5 \cdot (-3), 5 \cdot 4 \rangle = \langle -15, 20 \rangle$$

$$\langle 6, 24 \rangle + \langle -15, 20 \rangle = \langle -9, 44 \rangle$$

IV. **Unit Vectors:** have a magnitude or length of 1. They are useful in many applications of vectors.

$$u = \text{unit vector} = \frac{\vec{v}}{\|\vec{v}\|}$$

Ex 4) Find a unit vector in the direction of the given vector. $v = \langle -2, 5 \rangle$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle -2, 5 \rangle}{\sqrt{(-2)^2 + (5)^2}} = \frac{\langle -2, 5 \rangle}{\sqrt{4 + 25}} = \left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$$