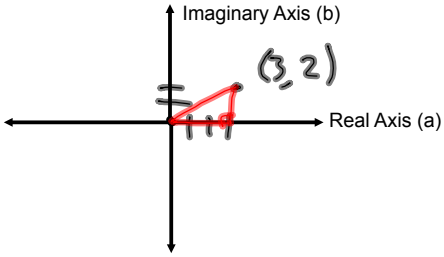
6.5 Trigonometric Form of a Complex Number
Day 1
**a unit circle is very helpful

I. **The Complex Plane**-- a complex number z = a + bi, (a, b) is a point.



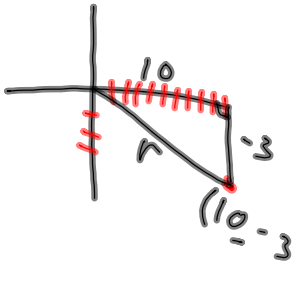
3 + 2i

II. Absolute Value of a Complex Number: z = a + bi

absolute value = | a + bi | = $\sqrt{(a^2 + b^2)}$

Ex 1) | 10 - 3i |

$r = \sqrt{a^2 + b^2}$



$r = \sqrt{(10)^2 + (-3)^2}$

$r = \sqrt{100 + 9}$

$r = \sqrt{109}$

III. **Trig Form of a Complex Number**: z = a + bi

z = r(cos θ + isinθ) a = rcosθ, and b = rsinθ

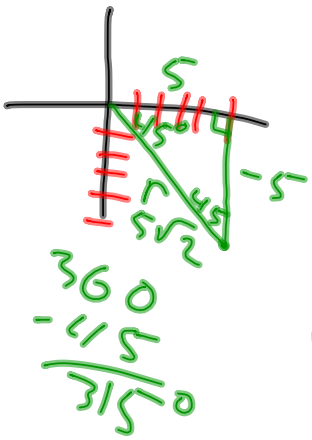
Ex 1) 5 - 5i

$r = \sqrt{a^2 + b^2}$

$r = 5\sqrt{2}$

θ is in standard position.

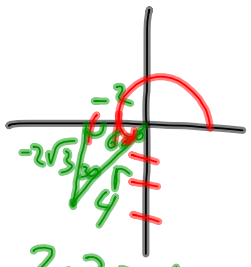
$\theta = 315^\circ$ or $\frac{7\pi}{4}$



360
- 45
315°

$z = 5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

Ex 2) $-2 - 2\sqrt{3}i$



$2 \cdot 2 = 4$

$r = 4$

$r = \sqrt{a^2 + b^2}$

$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$\theta = 240^\circ$ or $\frac{4\pi}{3}$

$z = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

IV. Finding the Standard Form of a Complex Number

Ex 3) $2(\cos 120^\circ + i\sin 120^\circ)$

$2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + \sqrt{3}i$

V. Multiplication and Division of Complex Numbers

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

product: $z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

quotient: $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$

Ex 4) Find the product and quotient of the following two complex numbers.

$z_1 = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ $\quad z_2 = 8(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi)$

Leave answer in Trig Form.

$z_1 z_2 = 2 \cdot 8\left(\cos\left(\frac{2\pi}{3} + \frac{11\pi}{6}\right) + i\sin\left(\frac{2\pi}{3} + \frac{11\pi}{6}\right)\right)$

$\frac{4\pi}{6} + \frac{11\pi}{6} = \frac{15\pi}{6} = \frac{5\pi}{2} = 16\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$

$2\frac{1}{2}\pi$

$z_1 z_2 = 16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Ex 4) Find the product and quotient of the following two complex numbers.

$z_1 = 2\left(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi\right)$ $z_2 = 8\left(\cos \frac{11}{6}\pi + i \sin \frac{11}{6}\pi\right)$

$$\frac{z_1}{z_2} = \frac{2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)}{8\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)}$$

$$\frac{z_1}{z_2} = \frac{1}{4}\left(\cos -\frac{7\pi}{6} + i \sin -\frac{7\pi}{6}\right)$$

$\frac{2\pi}{3} - \frac{11\pi}{6}$

$\frac{4\pi}{6} - \frac{11\pi}{6} = -\frac{7\pi}{6}$