6-5 Day 2

I. **DeMoivre's Theorem**: $z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$

Write in standard form.

Ex 1) $(-1 + \sqrt{3}i)^{12}$

$r = 2$

$\theta = 120°$

$n = 12$

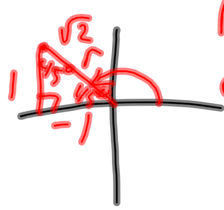
$\left[2(\cos 120° + i\sin 120°)\right]^{12}$

$= 2^{12}(\cos 12\cdot 120 + i\sin 12\cdot 120)$

$= 4096(1 + 0i)$

$\boxed{= 4096}$

Ex 2) $(-1 + i)^{10}$



$r = \sqrt{2}$

$\theta = 135°$

$z^n = \left[\sqrt{2}(\cos 135° + i\sin 135°)\right]^n$

$z^{10} = \left[\sqrt{2}(\cos 10\cdot 135° + i\sin 10\cdot 135°)\right]^{10}$

$z^{10} = (\sqrt{2})^{10}(0 + -1i)$

$= 32(0 + -1i)$

$= -32i$

II. **nth roots of a complex number**: z = r(cosθ + isinθ) has exactly n distinct roots given by :

$$\sqrt[n]{r}\left(\cos\frac{\theta + \overset{360^\circ}{2\pi} k}{n} + i\sin\frac{\theta + \overset{360^\circ}{2\pi} k}{n}\right) \text{ where } k = 0, 1, 2, \ldots, n-1$$

Ex 3) Find the three cube roots of z = -2 + 2i.

$n = 3$

$r = 2\sqrt{2}$

$\theta = 135^\circ$

$z = 2\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$

$K = 0 \Rightarrow \sqrt[3]{2\sqrt{2}}\left(\cos\frac{135 + 360\cdot 0}{3} + i\sin\frac{135+360\cdot 0}{3}\right)$

$(2\cdot 2^{\frac{1}{2}})^{\frac{1}{3}}$

$2^{\frac{1}{3}}\cdot(2^{\frac{1}{2}})^{\frac{1}{3}}$

$2^{\frac{1}{3}}\cdot 2^{\frac{1}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$

$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$

$\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= 1 + i$

$K = 1 \Rightarrow \sqrt{2}\left(\cos\frac{135+360\cdot 1}{3} + i\sin\frac{135+360\cdot 1}{3}\right)$

$\sqrt{2}(\cos 165^\circ + i\sin 165^\circ)$

$-1.366 + 0.366i$

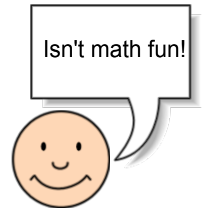
$K = 2 \Rightarrow \sqrt{2}\left(\cos\frac{135+360\cdot 2}{3} + i\sin\frac{135+360\cdot 2}{3}\right)$

$\sqrt{2}(\cos 285^\circ + i\sin 285^\circ)$

$0.366 + -1.366i$

$\sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$ where k = 0, 1, 2, ..., n-1

Let us now do #95 in your homework together!



$z = 16\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$n = 4,\ r = 16,\ \theta = 240°$

$k = 0 \Rightarrow \sqrt[4]{16}\left(\cos\frac{240+360\cdot 0}{4} + i\sin\frac{240+360\cdot 0}{4}\right)$

$2(\cos 60° + i\sin 60°)$

$2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$1 + \sqrt{3}i$

$k = 1 \Rightarrow \boxed{-\sqrt{3} + i}$

$k = 2 \Rightarrow \boxed{-1 - \sqrt{3}i}$

$k = 3 \Rightarrow \boxed{\sqrt{3} - i}$