7-4 Matrices and Systems of Equations

Day 1

I. **Order of a Matrix**: Rows x Columns

Ex 1) $\begin{bmatrix} 2 \end{bmatrix}$ 1 x 1

Ex 2) $\begin{bmatrix} 2 & 3 & 6 \\ 5 & 8 & 1 \end{bmatrix}$ 2 x 3

Ex 3) $\begin{bmatrix} 1 & 5 & 6 & 3 \\ 2 & 4 & 8 & 9 \\ -1 & 2 & 0 & 1 \end{bmatrix}$ 3 x 4

II. **Augmented Matrices**

Ex 5) Write the augmented matrix.

x + 3y = 9
-y + 4z = -2
x - 5z = 0

x y z

$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 9 \\ 0 & -1 & 4 & -2 \\ 1 & 0 & -5 & 0 \end{array}\right]$

Ex 6) Write the equations given the augmented matrix.

$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 2 & -3 & 4 \end{array}\right]$

$\begin{cases} x + 2y = 7 \\ 2x - 3y = 4 \end{cases}$

III. **Elementary Row Operations**...Let us read page 491, and then take a look at example 3. You need to be able to tell the difference between Row-Echelon Form and Reduced Row-Echelon Form...see page 493. Discuss Example 5 in book.

Ex 7) Use Gaussian Elimination with an augmented matrix to solve the system.

x - 2y + 3z = 9
-x + 3y = -4
2x - 5y + 5z = 17

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array}\right]$

$\xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right] \rightarrow$

$\xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{2R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 9 & 19 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

Row echelon Form

$\xrightarrow{-3R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 9 & 19 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{-9R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

1 way

reduced row echelon form

x = 1
y = -1
z = 2

(1, -1, 2)

2nd way

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$ → x - 2y + 3z = 9 → y + 3z = 5

row echelon Form

z = 2

y + 3(2) = 5

y = -1

x - 2(-1) + 3(2) = 9

x + 2 + 6 = 9

x = 1

(1, -1, 2)

Ex 8) $x - y + 2z = 4$
$x + z = 6$
$2x - 3y + 5z = 4$
$3x + 2y - z = 1$

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 1 & 0 & 1 & 6 \\ 2 & -3 & 5 & 4 \\ 3 & 2 & -1 & 1 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-1R_1+R_2} \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & -4 \\ 0 & 5 & -7 & -11 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix} \quad R_2 + R_3$$

$0 \neq -2$
no solutions

Ex 9) $2x + 4y - 2z = 0$
$3x + 5y = 1$

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 3 & 1 \end{bmatrix} \xrightarrow[-3R_1+R_2]{} \xrightarrow{-R_2} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$y - 3z = -1$
$y = 3z - 1$

$(-5z+2, 3z-1, z)$

$x + 2y - z = 0$
$x + 2(3z-1) - z = 0$
$x + 6z - 2 - z = 0$
$x - 2 + 5z = 0$
$x = -5z + 2$