7-6 The Inverse of a Square Matrix

I. **Inverse of a Square Matrix**: A must be a nxn. $A \cdot A^{-1} = I_n = A^{-1} \cdot A$
(A^{-1} is read "A inverse")

Ex 1) Show that B is the inverse of A, where $A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

$$BA = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

$\therefore$ B is the inverse of A.

Important:
Not all square matrices have inverses.
AB will most likely never be equal to BA.
If a matrix A has an inverse, A is called invertible or nonsingular.

II. **Inverse of a 2 X 2.**

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Ex 2) Find the inverse of $A = \begin{bmatrix} 2 & -4 \\ 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{16 - -16} \begin{bmatrix} 8 & 4 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{32} \begin{bmatrix} 8 & 4 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{8} \\ -\frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

III. **Use a Matrix Equation to solve the system**. $x = A^{-1} \cdot B$

$(\frac{1}{2})\, 2x = 10(\cdot\frac{1}{2})$

Ex 3) $2x + 3y + z = -1$
$3x + 3y + z = 1$
$2x + 4y + z = -2$

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

$3 \times 3 \quad 3 \times$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1+1+0 \\ 1+0+-2 \\ -6+-2+6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

$\boxed{(2, -1, -2)}$