

8-1 Sequences and Series

Ex 1) Write the first 4 terms. Assume n begins with 1.

$$a_n = 3 + (-1)^n$$

$$\begin{aligned} n=1 &\Rightarrow a_1 = 3 + (-1)^1 = 2 \\ n=2 &\Rightarrow a_2 = 3 + (-1)^2 = 4 \\ n=3 &\Rightarrow a_3 = 3 + (-1)^3 = 2 \\ n=4 &\Rightarrow a_4 = 3 + (-1)^4 = 4 \end{aligned}$$

Ex 2) Find the a_{25} term if $a_n = (-1)^n (3n-2)$

$$\begin{aligned} a_{25} &= (-1)^{25} [3(25) - 2] \\ a_{25} &= -1(75-2) \\ a_{25} &= -73 \end{aligned}$$

Ex 3) Write an expression for the apparent nth term. Assume n begins with 1. (Explicit formula)

A) 1, 4, 7, 10, 13, ...

n	1	2	3	4	5
a_n	1	4	7	10	13

$\begin{matrix} \times 3 \\ - 2 \end{matrix}$

$$a_n = 3n - 2$$

B) 2, 3, 4, 5, 6, ...

3 4 5 6 7

n	1	2	3	4	5
a_n	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$

$$a_n = \frac{n+1}{n+2}$$

Recursive Formulas: You need one or more of the first few terms. All other terms are defined using previous terms.

Ex 4) Find the first 5 terms if $a_1 = 28$ and $a_{k+1} = a_k - 4$.

$$\begin{aligned}
 a_2 &\Rightarrow k=1 \Rightarrow a_{1+1} = a_1 - 4 = 28 - 4 = 24 = a_2 \\
 a_3 &\Rightarrow k=2 \Rightarrow a_{2+1} = a_2 - 4 = 24 - 4 = 20 = a_3 \\
 a_4 &= 16 \\
 a_5 &= 12
 \end{aligned}$$

Simplify Factorials

Ex 5) $\frac{12!}{4!8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \dots}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \dots} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 3 \cdot 11 \cdot 5 \cdot 3 = 495$

Summation Notation:

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$$

Find the sum.

Ex 6) $\sum_{i=1}^4 3i = 3(1) + 3(2) + 3(3) + 3(4) = 30$

Ex 7) $\sum_{k=0}^3 \frac{1}{k^2 + 1} = \frac{1}{0^2 + 1} + \frac{1}{1^2 + 1} + \frac{1}{2^2 + 1} + \frac{1}{3^2 + 1} = 1.8$

Ex 8) Use sigma notation to rewrite.

$$\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \dots + \frac{1}{3(9)} = \sum_{k=1}^9 \frac{1}{3k}$$

Can use the calculator to find the sum:

Sum(seq(formula, x, lower limit, upper limit))

Sum—found under list-math-sum

Seq—found under list-ops-seq