8-5 The Binomial Theorem

Pascal's Triangle

n0 1 row 0

n1 1 1 row 1

n2 1 2 1 row 2

1 3 3 1 row 3

1 4 6 4 1

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

Binomial Theorem: $(x+y)^n$---expand

$(x+y)^n = x^n + nx^{n-1} + \ldots + \boxed{{}_nC_r X^{n-r} Y^r} + \ldots + nxy^{n-1} + y^n$

the coefficient of $x^{n-r} y^r$ is: $nCr = \dfrac{n!}{(n-r)!\,r!}$

${}_nC_r$

Ex 1) Find the binomial coefficient.

A) ${}_8C_2 =$

n r

$= \dfrac{8!}{6!\,2!} = \dfrac{8\cdot 7\cdot \cancel{6 \cdots}}{\cancel{6\cdot 5 \cdots}\cdot 2\cdot 1} = 28$

B) $\binom{10}{3} = {}_{10}C_3 = 10$ $\boxed{\text{math}}$ $\boxed{\text{prb}}$ $\boxed{{}_nC_r}$ 3

$= 120$

C) ${}_7C_0$

$= 1$

D) $\binom{8}{8} = 1$

Ex 2) Use a grapher to find ${}_{17}C_4$. = 2380

Ex 3) Use Pascal's Triangle to find the binomial coefficient for ${}_7C_4$.

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row term 5th term

r is one less than term # in pascal's △

Ex 4) Use the binomial theorem to expand and simplify $(3r + 2s)^6$.

x y n

row 6: 1 6 15 20 15 6 1

$1(3r)^6 + 6(3r)^5(2s)^1 + 15(3r)^4(2s)^2 + 20(3r)^3(2s)^3 + 15(3r)^2(2s)^4 + 6(3r)(2s)^5 + 1(2s)^6$

$= 729r^6 + 2916r^5s + 4860r^4s^2 + 4320r^3s^3 + 2160r^2s^4 + 576rs^5 + 64s^6$

Ex 5) Use Pascal's Triangle to expand the binomial $(2x-3)^4$. $\to n$

row 4: 1 4 6 4 1

$1(2x)^4 + 4(2x)^3(-3)^1 + 6(2x)^2(-3)^2 + 4(2x)^1(-3)^3$
$+1(-3)^4 = 16x^4 - 96x^3 + 216x^2 - 216x + 81$

Ex 6) Find the sixth term of $(a+2b)^8$.

$n=8$
$r=5$
$x=a$
$y=2b$

$_nC_r\, x^{n-r} y^r$

$_8C_5 \cdot a^{8-5}(2b)^5$

$= 56a^3\, 32b^5$

$= 1792a^3b^5$

Ex 7) Find the coefficient of the term a^6b^5 in the expansion of $(2a-5b)^{11}$.

$n=11$
$r=5$
$x=2a$
$y=-5b$

$_nC_r\, x^{n-r} y^r$

$_{11}C_5 (2a)^{11-5} \cdot (-5b)^5$

$462 \cdot 64a^6 \cdot -3125b^5$

$-92{,}400{,}000$