9-7 Graphs of Polar Equations Day 1

Types of Polar Graphs—see page 687

Testing for Symmetry in Polar Coordinates

The graph of a polar equation is symmetric with respect to the following if the given substitution yields an equivalent equation.

1. The line $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$
2. The Polar Axis: Replace (r, θ) by $(-r, \pi - \theta)$ or $(r, -\theta)$
3. The Pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$

Test for Symmetry. You will need page 340 or identity sheet.

Ex 1) $r = 10 + 4\cos\theta$

Graph, check, prove

$(r, -\theta) \Rightarrow$

$r = 10 + 4\cos(-\theta)$

$r = 10 + 4\cos\theta$

$\therefore$ has symmetry about the polar axis

Ex 2) $r = \dfrac{4}{1 + \sin\theta}$

$(-r, -\theta)$ or $(r, \pi - \theta)$

we think $\theta = \frac{\pi}{2}$

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$(r, \pi - \theta)$

$r = \dfrac{4}{1 + \sin(\pi - \theta)}$

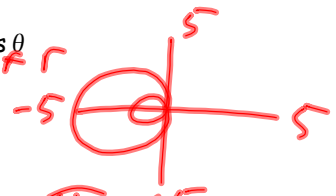
$r = \dfrac{4}{1 + \sin\pi\cos\theta - \cos\pi\sin\theta}$ ($\sin\pi \to 0$, $\cos\pi \to -1$)

$r = \dfrac{4}{1 + 0(\cos\theta) - -1\sin\theta}$

$r = \dfrac{4}{1 + \sin\theta}$ ∴ has symmetry about $\theta = \frac{\pi}{2}$

Ex 3) Find the maximum r-value for $r = 1 - 2\cos\theta$

Graph it and zeros of r



r max: $r = 3$ at $\theta = \pi$

$r = 0$: $0 = 1 - 2\cos\theta$

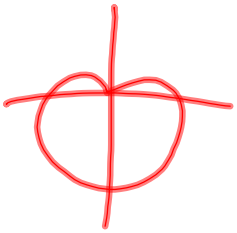
$-1 = -2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \cos^{-1}\frac{1}{2}$

$\theta \approx 1.047\ldots$ or $\theta = \frac{\pi}{3}$

Ex 4) Find the maximum r and zeros of r if $r = 10 - 10\sin\theta$



r_{max} is 20 at $\theta = \frac{3\pi}{2}$

zeros

$r = 0: \quad 0 = 10 - 10\sin\theta$

$-10 = -10\sin\theta$

$\frac{-10}{-10} = \frac{-10\sin\theta}{-10}$

$1 = \sin\theta$

$\theta = \sin^{-1} 1$

$\theta = 1.57$ or $\frac{\pi}{2}$