Chapter 11 Assignment List

11.1 Day 1: 3, 4, 7, 8, 9, 10, 12, 14, 16, 17, 18, 19, 20, 21, 22
Day 2: 24, 26, 28, 30, 32, 34, 40, 42, 44, 46, 50, 52

11.2 Day 1: 2, 4, 6, 8, 10, 14, 16, 20, 22, 24, 26, 28, 32, 38
Day 2: 40, 42, 44, 46, 48, 59, 64, 67, 68, 72

11.3 Day 1: 1, 2, 6, 8, 10, 14, 16, 20, 22
Day 2: 25, 26, 28, 29, 30, 32, 33, 36, 43, 46, 49, 52

QUIZ

11.4 Day 1: 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 29, 30
Day 2: 33, 34, 36, 38, 39, 40, 42, 49, 51, 52, 53, 54

11.5 Day 1: 2, 3, 4, 6, 8, 9, 10, 15, 16, 18, 19, 20
Day 2: 24, 25, 26, 28, 30, 32, 33, 34

Rev

TEST

11.1 Introduction to Limits
Day 1

The notion of a **limit** is a fundamental concept of Calculus.

Ex 1) You have 24 inches of wire and are asked to form a rectangle whose area is as large as possible. What dimensions should the rectangle have?

Using Limit terminology, we can say that "the limit of A as w approaches 6 is 36 and is written:

$$\lim_{w \to 6} A = \lim_{w \to 6} (12w - w^2) = 36$$

Definition of a Limit: If f(x) becomes arbitrarily close to a unique L as x approaches c from either side, the limit of f(x) as x approaches c is L, written $\lim_{x \to c} f(x) = L$

Ex 2) Estimate the limit numerically. $\lim_{x \to c} (3x - 2)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
y							

Ex 3) $\lim_{x \to 0} \frac{x}{\sqrt{(x+1)} - 1}$

x	-.01	-.001	-.0001	0	.0001	.001	.01
y							

Graph and find the limit.

Ex 4) $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x - 1}$

Ex 5) Find the limit of f(x) as x approaches 3, where f is defined as $f(x) = \begin{cases} 2, & x \neq 3 \\ 0, & x = 3 \end{cases}$

Ex 6) $\lim\limits_{x \to 0} \dfrac{|x|}{x}$

Ex 7) $\lim\limits_{x \to 0} \dfrac{1}{x^2}$

Ex 8) $\lim\limits_{x \to 0} \sin(1/x)$

x	
y	

Conditions under which Limits Do Not Exist

The limit of f(x) as $x \to c$ does not exist if any of the following conditions are true.

1. f(x) approaches a different number from the right side of c than it apporaches from the left side.

2. f(x) increases or decreases without bound as x approaches c.

3. f(x) oscillates between two fixed values of x approaches c.

11-1 day 2

Basic Limits on page 747.

1. $\lim_{x \to c} b = b$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} x^n = c^n$
4. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$

Properties of Limits on Page 747

1. Scalar Multiple:
2. Sum or Difference:
3. Product:
4. Quotient:
5. Power:

Direct Substitution and Properties of Limits

Find each limit.

A) $\lim_{x \to 4} x^2$

B) $\lim_{x \to 4} 5x$

C) $\lim_{x \to 9} \sqrt{x}$

D) $\lim_{x \to \pi} x \cos x$

Limits of Polynomial and Rational Functions

1. p = polynomial, c = real # $\lim_{x \to c} p(x) = p(c)$

2. r = rational function

$r(x) = \frac{p(x)}{q(x)}$ c is a real number $\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{g(c)}$

A) $\lim_{x \to -1} (x^2 + x - 6)$

B) $\lim_{x \to -1} \frac{x^2 + x - 6}{x + 3}$

11-2 Techniques for Evaluating Limits
Day 1

Ex 1) $\lim_{x \to -3} \frac{(x^2 + x - 6)}{x + 3}$

*This works since both functions agree at all but a single number c.
*This technique should only be used when direct substitution produces zero in the numerator and denominator.

Ex 2) $\lim_{x \to 1} \frac{x - 1}{x^3 - x^2 + x - 1}$

II. **Rationalizing Technique:** Rationalize the numerator by multiplying the numerator and denominator by the conjugate of the numerator.

Ex 3) $\lim_{x \to 0} \frac{\sqrt{(x + 1)} - 1}{x}$

III. **Technology**: The dividing out and rationalizing techniques may not work well for finding limits of non-algebraic functions...more sophisticated analytic techniques are needed.

Ex 4) $\lim_{x \to 0} (1 + x)^{1/x}$

Ex 5) $\lim_{x \to 0} \frac{\sin x}{x}$

Day 2 on 11-2

One-sided Limits: $\lim_{x \to c^-} f(x) = L$ $\lim_{x \to c^+} f(x) = L$

Ex 6) Find the limit as x approaches 0 from left and x approaches 0 if $f(x) = \frac{2x}{x}$

Existence of a Limit
If f is a function and c and L are real numbers, then $\lim_{x \to c} f(x) = L$ if and only if both right and left limits exist and are equal to L.

Ex 7) Find the limit of f(x) as x approaches 1 if f(x) = $4 - x, x < 1$
$4x - x^2, x > 1$

Ex 8) Find the $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

A) $f(x) = 5 - 6x$

B) $f(x) = \sqrt{(x - 2)}$

11.3 The Tangent Line Problem

Day 1

Skip 33

Calculus is a branch of math that studies rates of change of functions.
(They do have applications in real life.)

*rates of change and slope...see page 763

There is a more precise method which we will be doing...

Definition for slope of a graph (page 765)
The slope m of the graph of f at the point (x, f(x)) is equal to the slope of its tangent line at (x, f(x)) and is given by

$$m = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Provided the limit exists.

Ex 1) Find the slope of the graph of $f(x) = x^2$ at the point (-2, 4).

Ex 2) Find the slope of $f(x) = -2x + 4$ using the definition.

Ex 3) Find a formula for the slope of $f(x) = x^2 + 1$. What is the slope at the points (-1, 2) and (2, 5)?

Day 2 on 11-3

Definition of a Derivative: This is the formula for the slope of the tangent line to the graph of f, provided the limit exists.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Ex 1) Find the derivative of $f(x) = 3x^2 - 2x$.

Ex 2) $f(x) = \sqrt{x}$. Find f ' (x) and the slope of the graph at points (1, 1) and (4, 2).

11-4 Limits at Infinity and Limits of Sequences
Day 1

Graph $y = f(x) = \frac{x + 1}{2x}$

$\lim_{x \to -\infty} f(x) = .5$ and $\lim_{x \to \infty} f(x) = .5$

These limits mean that the value of f(x) gets arbitrarily close to .5 as x decreases or increases without bound.

Definition of Limits at Infinity

$\lim_{x \to -\infty} f(x) = L_1$ and $\lim_{x \to \infty} f(x) = L_2$

Limits at Infinity

$\lim_{x \to \infty} \frac{1}{x^r} =$ $\lim_{x \to -\infty} \frac{1}{x^r} =$

Find the limit.

Ex 1) $\lim_{x \to \infty} (4 - \frac{3}{x^2})$

Ex 2) $\lim_{x \to \infty} \frac{10}{x^2}$

Ex 3) $\lim_{x \to \infty} \frac{3 + x}{3 - x}$

Ex 4) $\lim_{x \to -\infty} \frac{4x - 3}{2x + 1}$

Ex 5) $\lim_{x \to -\infty} \frac{3x^2 - 4}{1 - x^2}$

Ex 6) $\lim_{t \to \infty} \frac{1 - 2t + 6t^2}{5 + 3t - 4t^2}$

Day 2 on 11-4

Did you notice from yesterday...

If n < m

If n = m

If n > m

I. Limits of Sequences

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

$\lim_{n \to \infty} \frac{1}{2^n}$

II. Write the first 5 terms and find the limit. Assume n begins with 1.

Ex 1) $\lim_{n \to \infty} \frac{2n + 1}{n + 4}$

Ex 2) $\lim_{n \to \infty} \frac{2n + 1}{n^2 + 4}$

Ex 3) $\lim_{n \to \infty} \frac{2n^2 + 1}{4n^2}$

Ex 4) $\lim_{n \to \infty} \frac{(-1)^n}{n}$

Ex 5) $\lim_{n \to \infty} \frac{n^2}{3n + 2}$

11-5 The Area Problem
Day 1

Objective: Find limits of summations.
Use rectangles to approximate areas of plane regions
Use limits of summations to find areas of plane regions.

Summation Formulas and Properties (page 782)

1. $\sum_{i=1}^{n} c = c \cdot n$ c = constant

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

5. $\sum_{i=1}^{n} (a_i \pm b_i) =$ [ ]

6. $\sum_{i=1}^{n} ka_i =$ [ ]

Ex 1) $\sum_{i=1}^{60} 7$

Ex 2) $\sum_{i=1}^{200} i$

Ex 3) $\sum_{j=1}^{25} j^2 + j$

Ex 4) $\sum_{i=1}^{n} \frac{i^3}{n^4}$

a) Rewrite as a rational function.

b) Complete the table.

n	10^0	10^1	10^2	10^3
s(n)				

c) Find the $\lim_{n \to \infty} s(n) =$

Area of a Plane Region--see example 4, page 785. You can get a negative area when under x-axis, just change it to positive.

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f\left(a + \frac{(b-a)}{n} i \right) \frac{(b-a)}{n}$$

Things to remember:

Ex 5) See number 17 on page 788. Find the area using rectangles: $f(x) = (.25)x^3$

Ex 6) **See #21 on page 788**. Complete the table showing the approximate area of the region in the graph using n rectangles of equal length.

A) $f(x) = (1/9)x^3$

n	4	8	20	50
A				

11-5 day 2

I. Use the limit process to find the area of the region between the graph of the function and the x-axis over the given interval.

Ex 1) $f(x) = 3x - x^2$ $\left[1, 2\right]$

Ex 2) $f(x) = 4x + 1$ $\left[0, 1\right]$

Ex 3) $f(x) = 2 - x^2$ $\left[-1,\ 1\right]$

Pre-Calculus

Extra Credit--Ch. ~~10~~ 11 **Name__________________**

Tyler, Julian, and Nadja have been earning money designing and producing brochures and flyers for small businesses in their area. They plan to use the paper and ink they have on hand to fill some orders they have for publications. Unfortunately, they have more work then time and would like to know how many of each type of publication they should manufacture to maximize their profits.

They have 500 sheets of production paper. Single sided flyers are sold in 20 copy packets and need 20 sheets of paper per pack. Double sided brochures come in 10 packs and would need 10 sheets of paper. They have 60 units of ink, and on average use 1 unit for a pack of flyers and 3 per pack of brochures. All of the projects they have are new orders and will need to be both designed and printed. On average, it takes 2 hours to design and print a packet of flyers and 3 for a brochure. They feel they can allocate a total of 72 hours to this. If their profit on a flyer pack is $10, and is $20 on a brochure pack, how many of each should they make to maximize their profit?